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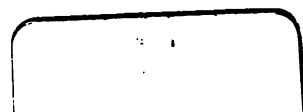
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THE  
PENDULOGRAPH



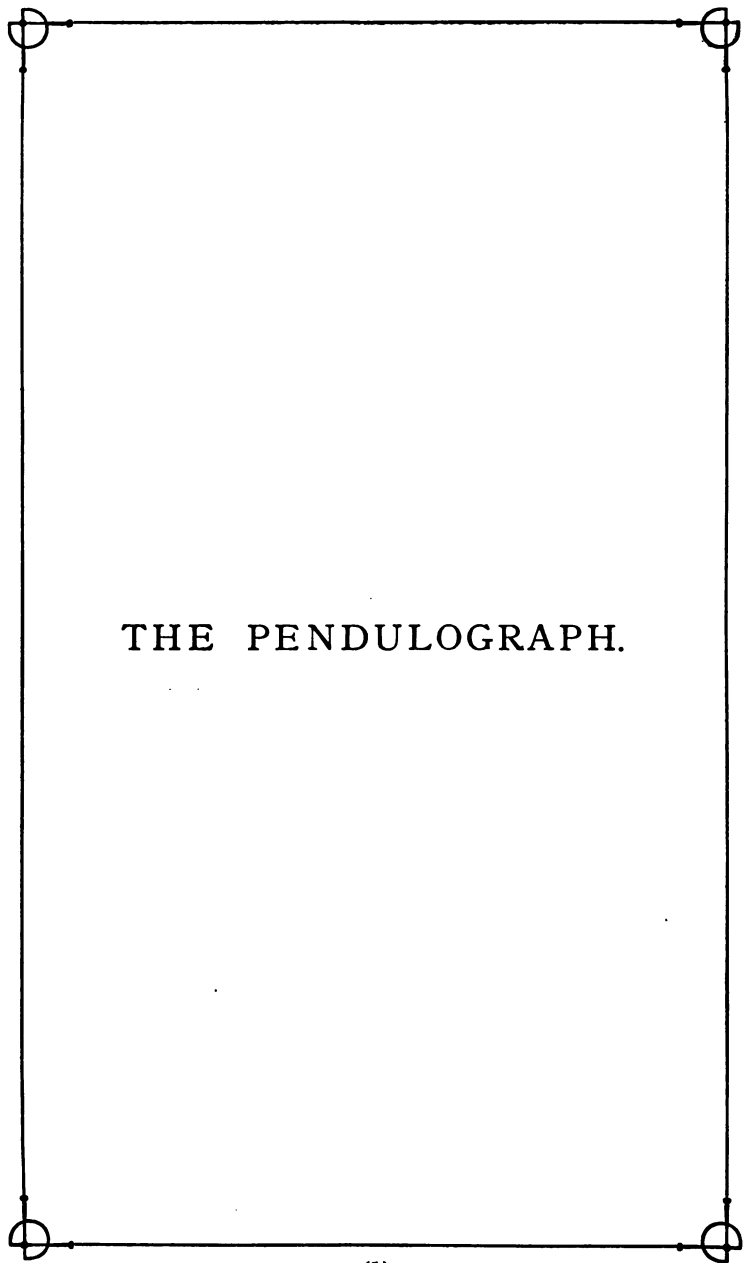
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πάντα διςσὰ ἐν κατέναντι ἐνός;  
καὶ οὐκ ἐποίησεν οὐδὲν ἐκλείπον.

ECCLESIASTICUS xlii. 24.

# THE PENDULOGRAPH:

A SERIES OF

**Bi-Pendulum Writings**

OF THE

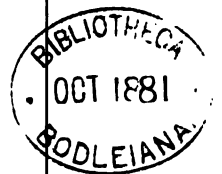
**TWENTY RATIOS OF THE MUSICAL SYSTEM;**

OR,

**SOUND SEEN IN THE SILENCE.**

BY THE

**REV. JOHN ANDREW.**



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*"A THING OF BEAUTY IS A JOY FOR EVER."*

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**London:**

**GEORGE BELL AND SONS,**

**YORK STREET, COVENT GARDEN.**

**1881.**

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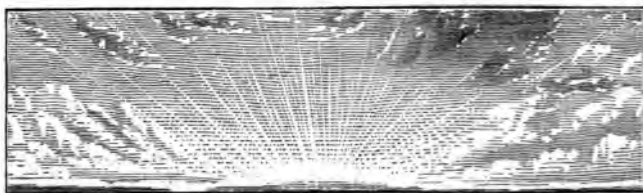
## Dedication.

UNCOUNSELLED ONE! whose Purpose to Create  
Upsprang within Thyself—upsprang in love,  
Which was its proper motion; ere as yet  
Was “The Beginning.” O Thou WORD of God!  
Revealer of that fontal Purpose: Thou  
Whose part it is to manifest, and make  
Intelligible; marshalling in hosts  
The forms Thou puttest upon all things made  
In heaven and earth, “even all the hosts of them,”  
Kingdom and Class, Orders and Genera  
Of Species, whose types indelible,  
Stereotypes, “whose seeds are in themselves  
After their kinds,” are all, O Lord, of Thee.  
THOU BROODING SPIRIT! “Lord and Giver of Life,”  
Whose quickening force setteth the quivering pulse  
In every living thing. O THREE-ONE GOD!  
I dedicate to Thee, from whom it came,

*McVicar's Cosmical Law—"Assimilation."*

This little finding of Thy beauteous ways.  
Thou art Thyself the Secret of Thy works.  
Thou art the Key. Thine image bear they all  
Or more or less. And heaven-born Music, as  
Thine ordinance in air and ear, and in  
The balance of the force elastic with  
The gravitating force that holdeth all,  
Music the statute is which more than most  
Of all that stands on Nature's statute-book,  
Image and superscription—Three in One  
In interlacing monogram, doth show  
Of Thee: to whom we render it *as due*.  
Accept the Tribute.





## TREATISE ON VIBRATIONS.

THE air which lies about this beautiful and busy planet of ours is full of various motions; it is so sensitively elastic and so easily agitated. Forces great and small continually fill it—now with windy urgings driving it along in currents, now with gentle musical strokes setting it a quivering to and fro. It is these quiverings, usually called “Vibrations,” that produce musical sounds; and they are produced not in the still air only, but in the rolling currents of the storm as well. Many instruments have been constructed by man—strings, pipes, bells, drums, &c., for the purpose of producing those musical vibrations of the air so pleasing to the ear.

A vibrating string crushes back and condenses the elastic particles of the air now on this side of it and now on that; and as the string returns from this side and from that, the crushed air speedily recovers itself, following the retiring string, only to be crushed back again and again by every vibration of the string.

Some teachers on the subject consider a *vibration of a string* as the passage from extreme to extreme of the motion; and some consider it as the passage from extreme to extreme and back. D. C. Ramsay, of Glasgow, has shown his pupils reason to consider a vibration as the movement from the

*How to reckon Vibrations : various views.*

string's centre of rest to the extreme on either side and back ; inasmuch as the condensation of the air on the right side of the string is that of one portion of air, and the condensation on the left side is that of a different portion. And still further, the rarefaction of the vibration of the right side is taking place simultaneously with the condensation of the vibration of the left side, and *vice versa* ; so that a vibration of a string, if viewed from extreme to extreme and back, is really a double vibration, or rather, we might say, a *pair of vibrations*, the one being from the string's centre of rest to the right extreme and back, and the other from the string's centre of rest to the left extreme and back. This pair of vibrations, as they repeat themselves on the right and left of the string's centre of rest, are sending off symmetrically on either side pairs of *vibrations of the air itself*. For as the string strikes out to the right, the air is crushed together before it ; this is called the condensation ; and as the string returns toward its central position, the elastic air springs back after it, being relieved from the pressure ; this is called the rarefaction of the vibration ; and the same thing is happening on the other side. The crushed layer of air has in the meantime communicated its impulse to another layer beyond it, which also becomes condensed for an instant ; but as the first layer is relieved from its crushed state by the retiring of the string, so the second also is relieved from its condensed condition by the retiring of the first layer ; and so on, layer after layer, out into the surrounding space as far as the stroke of the vibrating string has had force to penetrate. The impulse of every outward layer becomes fainter and feebler in its conflict with the force of gravitation ; but the vibration is *executed in the same time*, however faint in force. All that is happening on the one side of the string is happening symmetrically on the other. If the vibrating body

*Vibrations become Weaker, but not Slower.*

were a bell instead of a string, then it would send out four impulses in two pairs, arranged diametrically opposite to each other ; or if the bell should divide into six, or eight, or ten, or twelve vibrating sections, they would produce a corresponding number of vibrations and vibratory waves in the surrounding air, which should all behave themselves in the same way as those of the string which we have described. And so with all other sorts of musical vibrations. Dr. Stone very nicely says—"A bell, when rung in the open air, throws off spherical shells of alternate condensations and rarefactions."—*Basis*, p. 6. Curwen says—"In thinking of the voice, we must imagine each impulse to extend from the mouth upward, downward, forward, sideward, and in all directions, forming spheres of impulses."—*Musical Statics*, p. 4.





## THE GENESIS OF THE MUSICAL SCALE.

IN seeing THE GENESIS OF THE MUSICAL SCALE of notes we have only to do with the *rate of speed*, that is, with the *number of vibrations per second of time*; as to the *loudness* and *quality* of the notes, these are other matters.

By sliding the finger slowly up a violin string while bowing upon it, we have a gradually rising sound. We might stop at any point of the thousand points and sound the note steadily, and give it a name. It might happen to be no note of the concert-pitch scale; but it would be a note as truly and sweetly as any of them. Now supposing we had a stretched string long enough, when bowed upon, to vibrate as slowly as the seconds pendulum of a clock, of course it would produce no sound by such a rate of speed; but if we should slide the finger up till the rising rate of vibrations should be about sixteen times faster than at the first, it would begin to be heard as a very low sound, which, as the finger was still moved upward, would ascend in pitch by unrecordably small degrees, till at last, far up, where it should be vibrating let us say at the rate of between 30,000 and 40,000 vibrations for one of the silent vibrations by which it began, the sound should become so piercingly high that the ear would begin to lose hold of it; and the vibrations, still increasing in quickness by the still upward moving finger,

### *The Reign of Mathematical Law.*

would *cease to produce sound*, and become as silent to the common human ear as when it was only vibrating seconds. We began below the audible range of vibrations, and we have continued beyond and above it; and between the low silence and the high silence lies the whole musical range of sound.

Now we must cut this gradually ascending inflection of sound into parts, and make a musical scale of distinctly stepping notes. Where shall we begin, and how shall we proceed?

We may begin anywhere; for in this slide of sound we have thousands of points, at any one of which we might stand still and make a note. Suppose that we agree to begin at the point where the string is vibrating forty-three times in a second, and call it F; a very low F, not far above where the low silence ends and we begin to hear the vibrations. Now having made a stand, and having defined the number of vibrations for a *fundamental note*—having selected a *root* for the musical system of vibrations, *we have no further choice*; our further work in cutting the slide of sound into notes for the musical scale is *all under mathematical law*, to which the human ear renders a willing and grateful obedience. Musical tones must have a certain distance between them before they can be arranged in scale for melodious succession, or sounded together in harmonious concord. What that distance is has not been left to individual taste, it is under Nature's mathematical rule, to which the ear is perfectly subject, having been constituted for it by the same Creating Hand and Mind, as other faculties of man are constituted for the same law in other spheres of sense and reason. What these distances have been made by Nature's mathematical hand will be seen immediately when we set forth the ratios which measure them. We must measure

them to *know* them as Nature has measured them to *make* them ; and fortunately we have the same rule.

But since F, with forty-three vibrations per second, is to be our point of departure, and *the unit* of the mathematical ratios of the scale, let us, for the sake of simplicity, call it one instead of forty-three : this will reduce the ratios of the whole scale to their simplest expression, and make them most easily appreciable. This method of simplification is usually followed by all who have attempted to exhibit this matter. Whether the secret of Nature now revealed had been found is another question. In some things it has been a long time, even many centuries of observation and effort, before the true order of Nature has been discovered. Astronomy, Botany, Chemistry, may suffice as instances. Music, *on its intellectual side*, may be found to be another illustration of this long toil and failure ending at last in light and joy.

It requires a range of numbers, from one to sixty-four, to develop and enclose the musical series ; no less than six octaves being required for the *natural genetic development* of the notes of the octave scale ; which is a very highly artistic condensation, containing everything. But the "*arithmetical progression*" of numbers, usually supposed to be that of the musical scale, by its universally admitted failure, is evidently not the progression of numbers chosen by Nature for the System of Music. She has been elective, and has chosen certain primes for her foundation, and their multiples for her beautiful building. Out of the sixty-four numbers of the arithmetical series from one to sixty-four, only twenty-five are elected by the spirit of music, and thirty-nine are rejected : thus—1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 27, 30, 32, 36, 40, 45, 48, 54, 60, 64.

Neither is it Nature's method to *choose the simplest ratios* which lie within the limits of the octave, as—1,  $\frac{5}{4}$ ,  $\frac{4}{3}$ ,  $\frac{3}{2}$ ,  $\frac{5}{3}$ , 2,

### *Each Interval in Three Magnitudes.*

and then, finding gaps where a 2nd and a 7th should be, to equalise the intervals by interpolating a note in each place, *discovered on some other principle*. Nature does not patch her work in this way.

We shall now call to our aid another string, that we may proceed not only in the way of showing the notes that follow each other sweetly in melodious succession, but that we may consider them also as they combine harmoniously in concord.

When the vibrations of two strings are coincident, that is, one for one, the effect goes simply to increase the sound in amount; but it is the same sound in both; indeed they are scarcely observed as two at all, and are said to be in *unison*—the one string vibrating exactly 50 or 500 to 50 or 500 of the other, as the case may be. But when the one string begins to be tuned to a quicker rate and to leave the other, at first the effect is jarring and disagreeable, till the distance between is such that 27 of the slow string is made for 32 of the tuned one. This ratio produces a very interesting effect in the ear—spicy and exciting. It will be observed that  $27 : 32 :: 5\frac{2}{5} : 6\frac{2}{5}$ , and is on the way to and nearly at the goal of  $5 : 6$ , when instantly the effect becomes much softer and more melodious, though still spicy and stirring. The first is the *Small Third*, which lies between D and F; while the last is the *Medium Third*, which occurs in three places of the scale—between A and C, between E and G, and between B and D. There is yet *another third*, larger in interval, simpler in ratio, and softer in effect, which is vested in the ratio of  $4 : 5$ , and occurs also in three places of the scale, namely, between F and A, between C and E, and between G and B.

But we must proceed to show the Method of Nature in Generating the Notes, amongst which these and all the others of the nineteen ratios of the musical system are found.

If the number of vibrations of any musical note be multiplied by two, and by three, and by five, its *octave*, its *fifth*, and its *third* will be discovered, or, we may rather say, *developed from their root*. This is what is known as the common chord of the note. The celebrated mathematician, Leonard Euler, in his "Letters to a German Princess," published in French in 1768, thus develops the three notes of the common chord—"My intention," says he, "is to render visible the real origin of musical notes, with which musicians themselves are almost totally unacquainted." Euler had already, in 1739, published a work in Latin, "*Tentamen novæ Theoriæ Musicæ*," in which he first made "An Essay at a New Theory of Music." Nicholas Fuss, in his Eulogé of the works of Euler, says this work "contained too much geometry for the musicians, and too much music for the geometers." He develops the musical scale from the three mathematical primes, 2, 3, and 5. He says—"Such, then, is the real origin of the musical notes already in use; they are derived from the numbers 2, 3, and 5. Were we further to introduce the number 7, the number of the tones of the octave would be increased, and the art of music carried to a higher degree of perfection. But here the Mathematician gives up the Musician to the law of his ear." Now D. C. Ramsay, of Glasgow, in his forthcoming work on the "Intellectual Basis and Build of Music," will show that the musical scale is built up mathematically of three such chords, growing out of each other in the following simple and beautiful way. The genesis of the scale does not begin with C, the root of the tonic chord, and the key-note and centre of rest of every musical creation, but with F, the root of the sub-dominant chord—the fifth which stands below the tonic fifth. *This is the true genetic point of departure, if we would see the family relation of notes and chords in the musical scale.* It matters not what

### *The Chord Scale, Genetic Scale, and Octave Scale.*

key we take, for all keys are alike in this respect; see one, see all. The key of C is chosen simply for convenience.  $F^1$  multiplied by 2 gives the octave,  $F^2$ ; multiplied by 3 it gives the fifth,  $C^3$ ; and multiplied by 5 it gives the third,  $A^5$ . Thus arises the common chord—F, A, C, F. Now take  $C^3$ , the top of the sub-dominant fifth, and treat it in the same way as we have treated F; multiply by 2, 3, and 5.  $C^3$  multiplied by 2 gives  $C^6$ , its octave; multiplied by 3 it gives  $G^9$ , its fifth; and multiplied by 5 it gives  $E^{15}$ , its third. This is our second and central chord—C, E, G, C. Now take  $G^9$ , the top of this tonic chord, and again pursue the same mathematical method—multiply by 2, 3, and 5.  $G^9$  multiplied by 2 will give  $G^{18}$ , its octave; multiplied by 3 it will give  $D^{27}$ , its fifth; and multiplied by 5 it will give  $B^{45}$ , its third. Thus is generated the upper fifth of the key—G, B, D, G. And now we have all— $F^1$ ,  $A^5$ ,  $C^3$ ,  $F^2$ ;  $C^3$ ,  $E^{15}$ ,  $G^9$ ,  $C^6$ ;  $G^9$ ,  $B^{45}$ ,  $D^{27}$ ,  $G^{18}$ . Such is the *Chord Scale* of the musical system.

In order to see the *whole Genetic Scale*, set them in the order in which they arise, filling in the octaves, thus—

#### *The Genetic Scale.*

F	F	C	F	A	C	F	G	A	C	E	F	G
1	2	3	4	5	6	8	9	10	12	15	16	18
A	C	D	E	F	G	A	B	C	D	E	F	
20	24	27	30	32	36	40	45	48	54	60	64	

Behold the Genetic Scale! A trinity of chords; each having also a trinity of notes; each chord shut in by the octave of its root, to give its variety unity; and the whole shut in by the octave of the root of all, making one grand chord of chords!

And now observe, also, that at the top of the *Genetic Scale* we find the *Octave Scale* developed, as the wonderful

### *The Great Three-times-three-in-one Chord.*

blossom produced by this vigorous and symmetrical stalk ; and by a perfectly natural process of growth. Although the seven notes of the octave scale are here standing in a solid mass, yet if sounded all together, along with and supported by the roots from which they spring, the whole together make one magnificent chord—the Sum and Crown of Harmony. Of course the hands cannot strike this grand and extended chord, which reaches across the whole key-board of the piano ; but a thin board of wood, notched so as to put down all the notes at a stroke, will enable any one to produce and hear this grand chord, once heard never to be forgotten.

By reference to the Genetic Scale above, it will be seen that the first single-step-interval which arises in the generations is from  $F^8$  to  $G^9$  ; and the next is from  $G^9$  to  $A^{10}$  ; and the third one-step-interval is from  $E^{15}$  to  $F^{16}$ . Thus there are three seconds of three different magnitudes ; and whatever may be the number of vibrations actually producing them—that is, whatever may be the pitch at which the musical scale is set—these one-step-intervals will always be found to have the same ratios, 8 : 9, 9 : 10, and 15 : 16. This variety of the elementary intervals, skilfully mingled by Nature's art as it is in the Octave Scale, where every step of the scale differs in measure both from the one before it and from the one after it, causes a similar variety in all the larger steps, inasmuch as the larger intervals are made up of the smaller ones.\* F-A is the first double-step-interval, and has the ratio of 4 : 5 ; A-C is the next, with the ratio

\* Of course this remark does not refer to the *Genetic development* of the intervals. For it will be observed at a glance that the larger intervals are first developed ; the very first being the octave, which embraces all. We simply mean to say that the contents of the larger intervals are the smaller ones. It is a remarkable fact, that so incommensurable are the musical intervals, that neither the *Comma*, which is a little less than the *ninth* part of the large second, nor the *Skisma*, which is only the *eleventh* part of the *Comma*, can be used as a unit of measurement exactly to measure them.

### *How Music abounds in Trinities.*

of 5 : 6 ; and when we come to D-F, we find the ratio of 27 : 32, which will not reduce to a simpler expression. So we have *three Thirds* as well as *three Seconds* of different magnitudes. In the same way we find *three Fourths* : C-F has the ratio of 3 : 4 ; A-D the ratio of 20 : 27 ; and F-B the ratio of 32 : 45. Again, in *Fifths* we have the same variety : F-C has the ratio of 2 : 3 ; D-A that of 27 : 40 ; and B-F that of 45 : 64. These are the three types of *Fifths*. The three different *Sixths* are : C-A, ratio 3 : 5 ; A-F, ratio 5 : 8 ; and F-D, ratio 16 : 27. Lastly, in *Sevenths*, we find A-G, ratio 5 : 9 ; F-E, ratio 8 : 15 ; and G-F, ratio 9 : 16.

Such is the system—at least the Masculine Scale of the system—of musical vibrations, which D. C. Ramsay's forthcoming work will set forth on both its Masculine and Feminine sides. We abstain at present from touching the Feminine Scale, with its wonderful inverse development ; but we have been permitted by the author thus far in our own words to anticipate his work, in order to accompany with their mathematical rationale the Pendulum Pictures which we, in this little art-volume, present to the lovers of the beautiful, and to the students of Nature's analogical mysteries.





## THE PENDULOGRAPH.

WE have spoken of the "low silence" in which the vibrations of Pendulums are performed. In this "low silence," just as in the "high silence," where the vibrations are too quick to be heard, and in the audible range, which lies between them, the vibrations are still under the same laws. And in the "low silence" we can adjust pendulums to swing the same ratios as we now know to be the ratios of the intervals, concords, and discords which exist in the musical system of vibrations. It is in this "low silence," and by the use of pendulums, that we produce the beautiful and interesting PENDULOGRAPHS now put before the musical public. These Pendulographs are pictures or portraits of the intervals, concords, and discords of the Musical System. They are produced by a pen placed under the control of two pendulums, which are tuned to swing, in the "low silence," the ratios of the musical intervals. When the pendulums are tuned to any of the ratios, the pen is so placed between them as to be moved by a blend of their two various motions; and if the tuning and the mechanism of the instrument be perfect, and the motion smooth and equal, then the picture will be a true Pendulograph, or portrait of the interval, chord, or discord, and will have a beauty to the eye analogous to the melodious succession or harmonious combination in the ear of the two notes which the pendulums are tuned to swing.

D. R. Hay, of Edinburgh, in several art works of great interest, developed the analogy of Music and Form by *angular proportion*. That analogy is in lines that all return upon themselves, whether circle, square, parallelogram, or ellipse. The parallelogram of the letterpress of our present little work is an illustration of this kind of proportion. It is determined by the angle of  $30^\circ$ , which is to  $45^\circ$  as  $3:2$  of the fundamental angle of  $90^\circ$ , and corresponds to the chord of the fifth if compared with a square. The analogy exhibited in the Pendulograph is not in lines returning upon themselves, but in *spiral lines*.

Each Pendulograph is *one spiral line*, beginning with the outer stroke and lessening toward the centre, where the pen comes to a stand-still, corresponding to the silence of the strings as they cease to vibrate. The mechanism of the instrument requires to be very good; and the tuning of the pendulums is as nice in the *silence* as the tuning of two strings in the region of *sound*.

We have arranged the Pendulographs in pairs. First, the Unison and the Octave; and after these the other intervals and their inversions—the Seconds, and their inversions the Sevenths; the Thirds, and their inversions the Sixths; the Fifths, and their inversions the Fourths. This is a twofoldness, which Nature—according to a law of symmetry, in which she delights—has set in Music. A very old writer says—"All things are twofold, one over-against another, and He made nothing incomplete." Nature goes in pairs. It will be observed that these pairs of intervals divide the beauty between them. If one be very beautiful in its curves, it is at the expense of the other. The Second which has  $9:10$  for its ratio is one of formosal beauty; so, also, is the Second which has  $8:9$ ; but the Sevenths that are their inversions are spirals of a very intricate order—too intricate

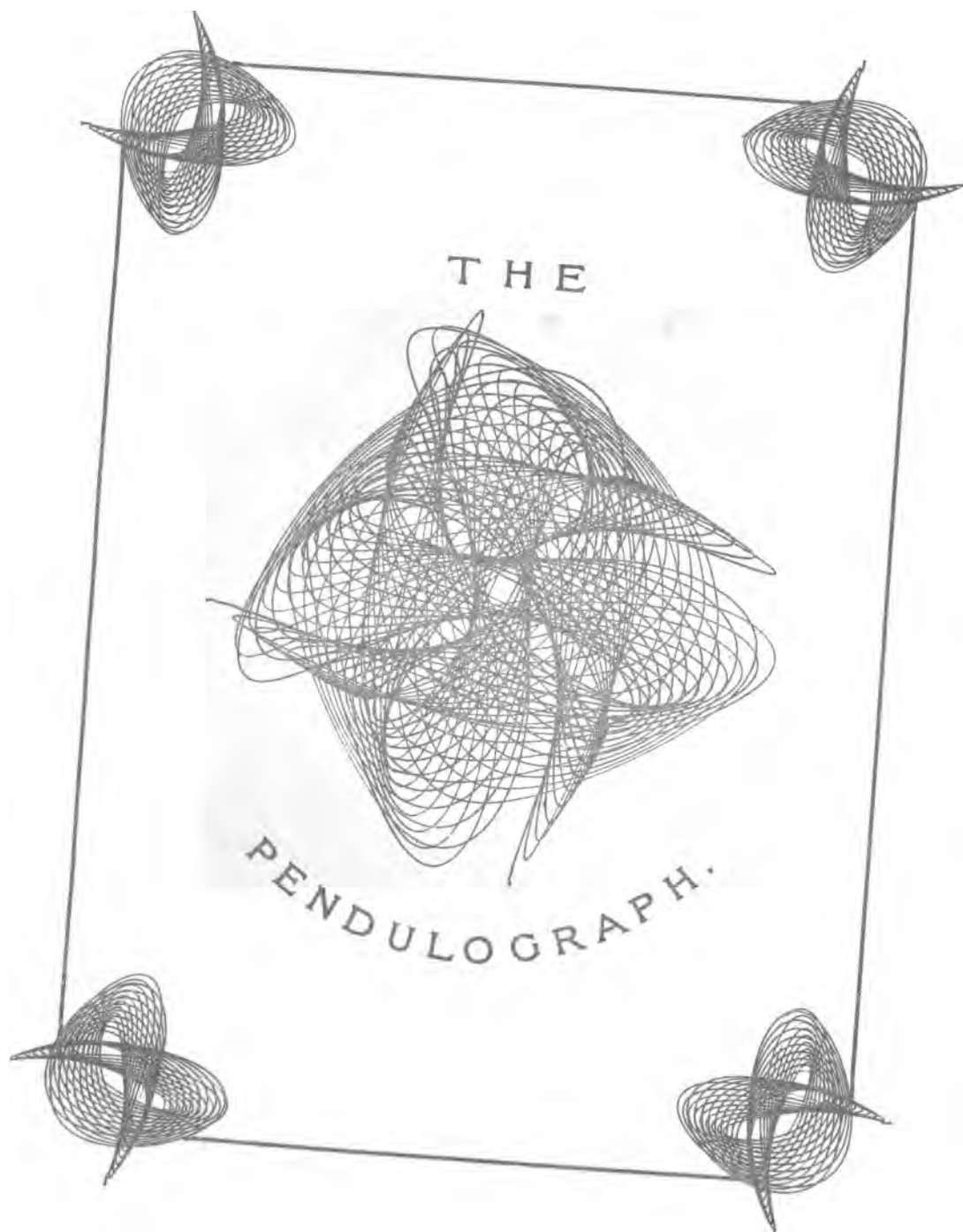
### *The Cosmical Law of Pairs.*

to be beautiful; while the Thirds which have 5 : 6 and 4 : 5 for their ratios, and which are of a very simple type of beauty, have for their inversions Sixths which are of a very peculiar and interestingly complex type. This fact may suggest something to the scientific student of musical sound. Meanwhile, we simply put forth these Musical Spirals for the consideration of the Musical world, with the assurance that they will find a place among those things of Beauty that are a joy for ever.

We have only further to add, that in the two Pendulums there is also a ceaseless source of amusement; for not only can we have multitudes of other ratios besides those which constitute the Musical Scale, but an *unequal pull* and an *unequal start* of the Pendulums will also produce an almost endless variety of exquisitely beautiful and many very curious forms.

Dr. McVicar, of Moffat, in his delightful science primer on "The Nature of Things," speaking of the molecular system of Nature, shows good reason to believe that the material element itself is "*alternative in its structure*;" and he says—"As soon as we enter on the detail of molecular synthesis, it will appear that this mode of union by coupling, or conjugation, is the very characteristic of that system, and the key of molecular construction and chemistry." From beginning to end Nature is "Two Witnesses" to the Creator. The author, loyal to this principle of two-foldness, deep-seated in the molecular "prelude to Sex," notably seen in Music, and desiring that his scientific tract should exhibit that highest type of the *Parallelism* which lies two-lined in the Science of the Soul and the Worship of the Spirit of man, closes, as he opens, with a sincere desire that his work may be—

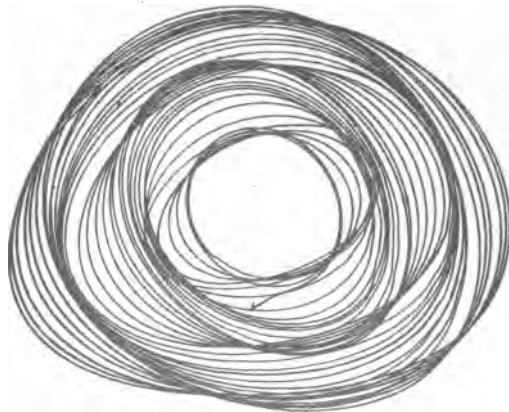
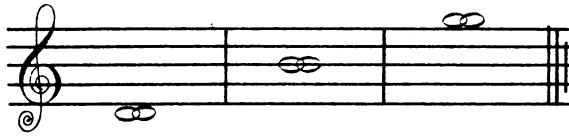
AD MAJOREM GLORIAM DEI.





I

THE PENDULOGRAPH.



## II

### THE PENDULOGRAPH.

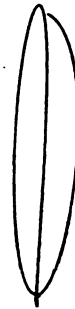
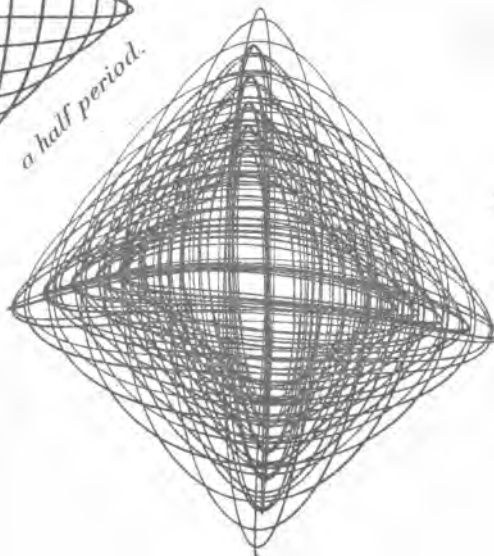
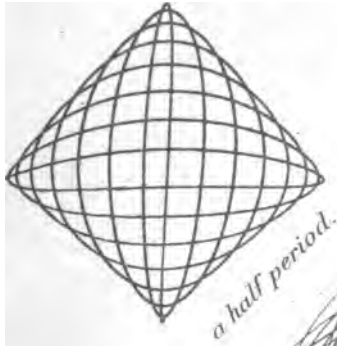
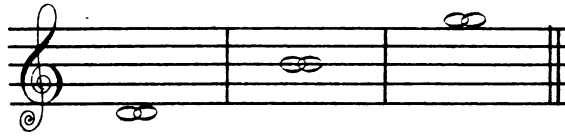


THE OCTAVE. Ratio 2:1.

*Number of Commas 5,5,8,8,9,9,9.*

### III

#### THE PENDULOGRAPH.

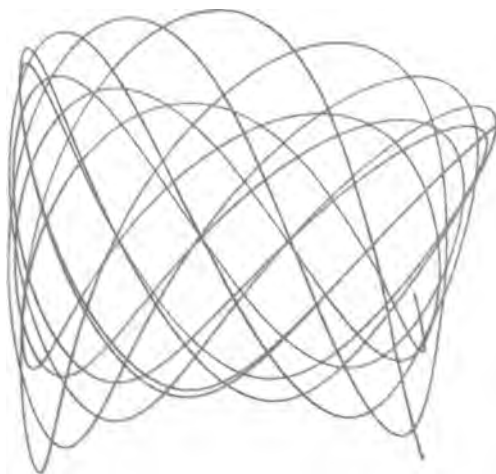


THE SMALL SECOND. Ratio 15:16.

*Number of Commas - 5.*

# IV

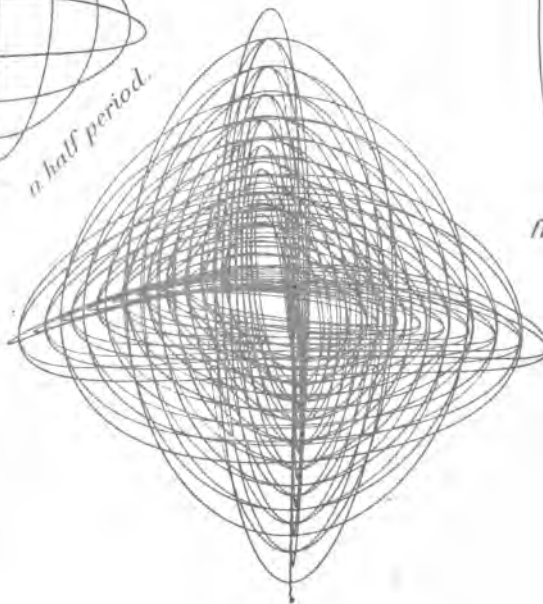
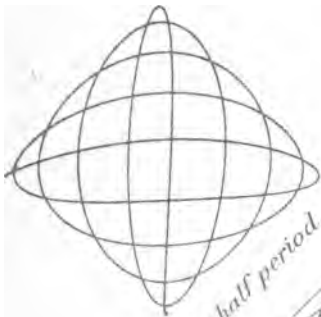
## THE PENDULOGRAPH.



THE GREAT SEVENTH. Ratio 8:15.

*Number of Commas 5,8,8,9,9,9.*

## THE PENDULOGRAPH.

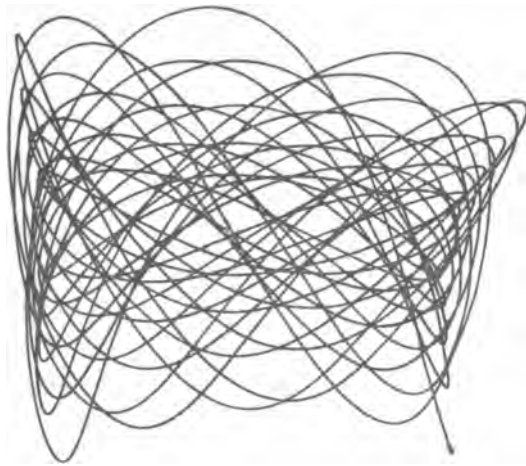


THE MEDIUM SECOND. Ratio 9:10.

*Number of Commas\_ 8.*

# VI

## THE PENDULOGRAPH.

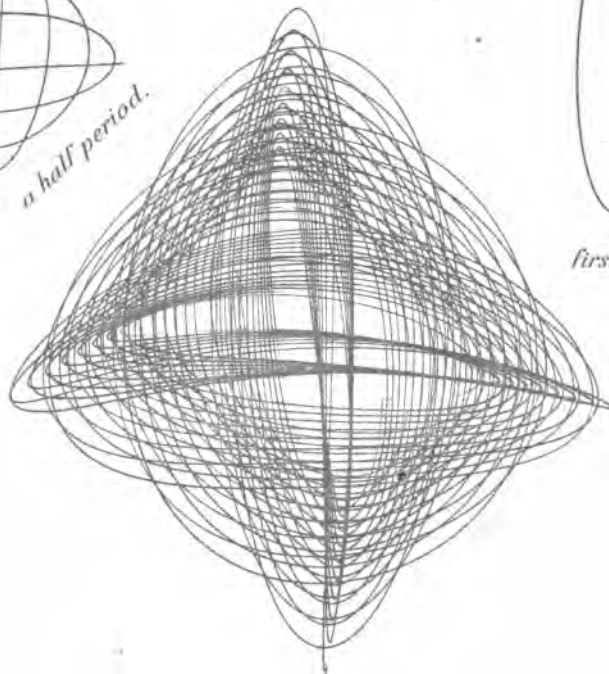
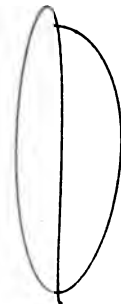
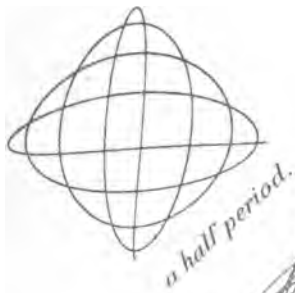


THE MEDIUM SEVENTH. Ratio 5:9.

*Number of Commas. 5,5,8,9,9,9.*

# VII

## THE PENDULOGRAPH.

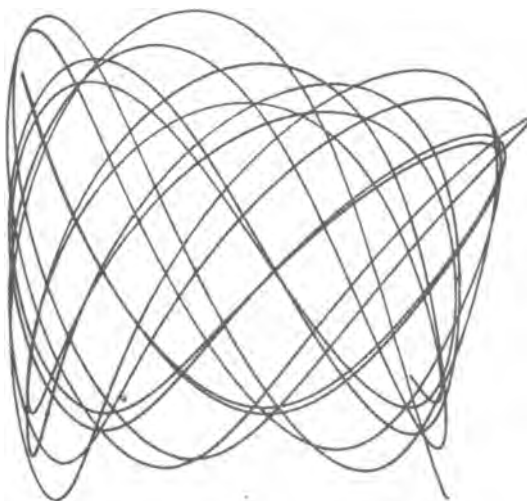


THE GREAT SECOND. Ratio 8:9.

*Number of Commas\_9.*

# VIII

## THE PENDULOGRAPH.

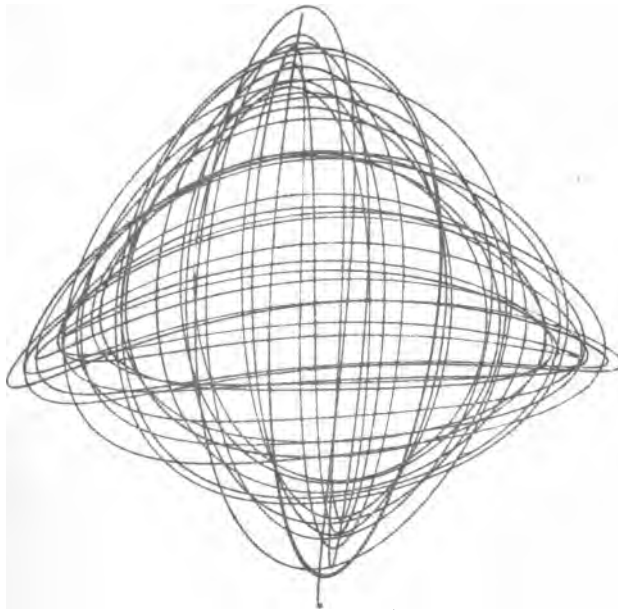


THE SMALL SEVENTH. Ratio 9:16.

*Number of Commas 5,5,8,8,9,9.*

# IX

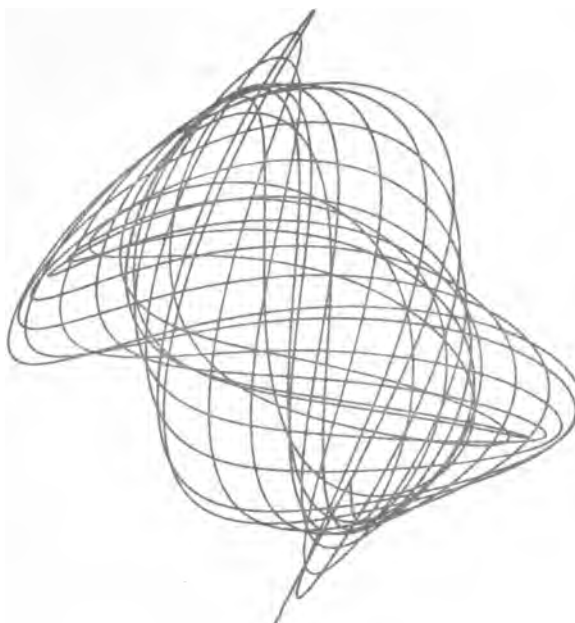
## THE PENDULOGRAPH.



THE SMALL THIRD. Ratio  $27:32$ .

*Number of Commas 5, 8.*

THE PENDULOGRAPH.

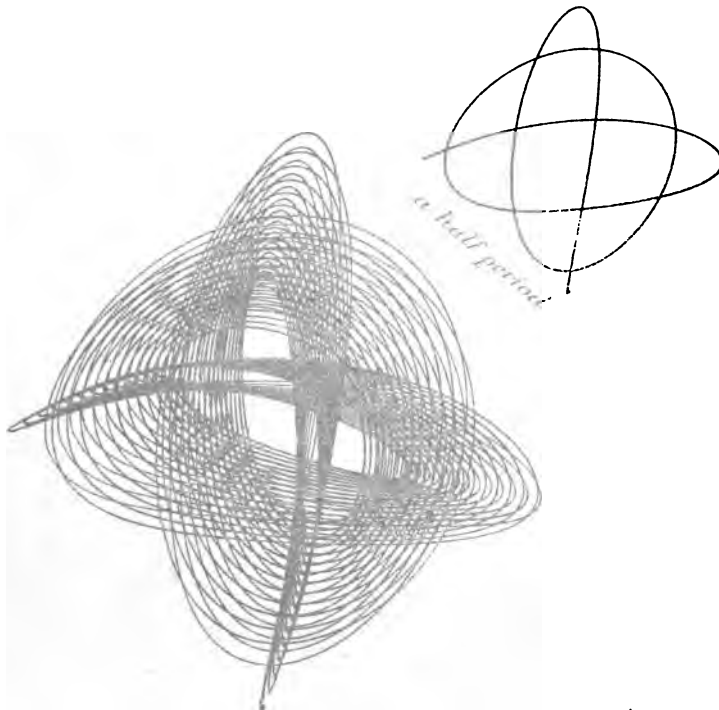


THE GREAT SIXTH. Ratio 16 : 27.

*Number of Commas 58,999.*

# XI

## THE PENDULOGRAPH.

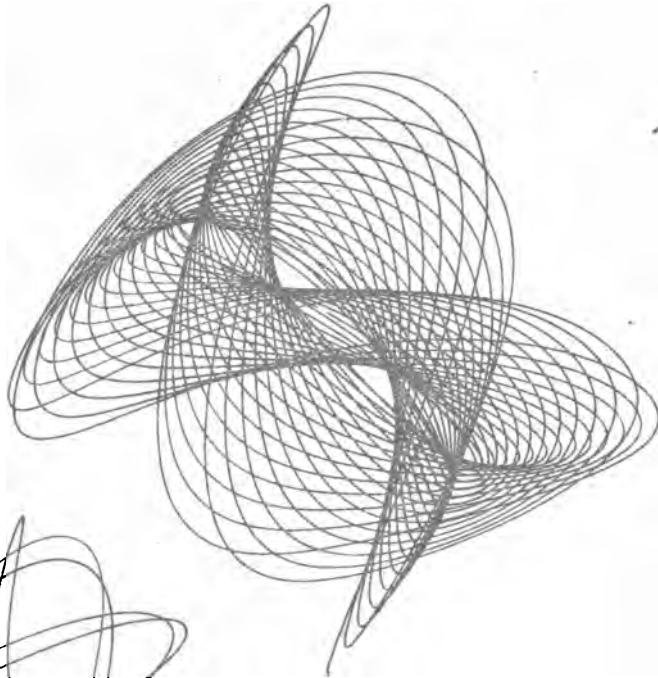


THE MEDIUM THIRD. Ratio 5:6.

*Number of Commas\_5,2.*

## XII

### THE PENDULOGRAPH.



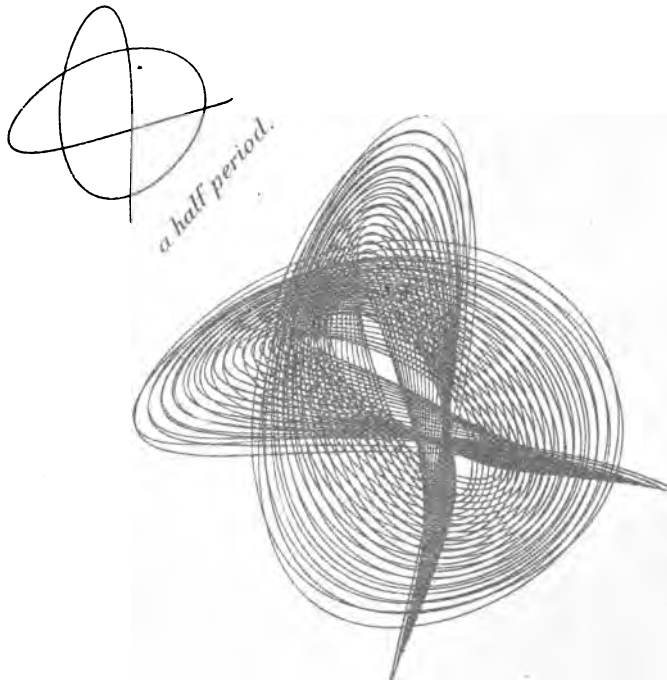
*a whole period.*

THE MEDIUM SIXTH. Ratio 3:5.

*Number of Commas 5,8,8,9,9.*

# XIII

## THE PENDULOGRAPH.

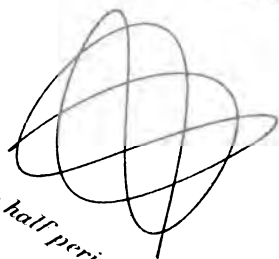
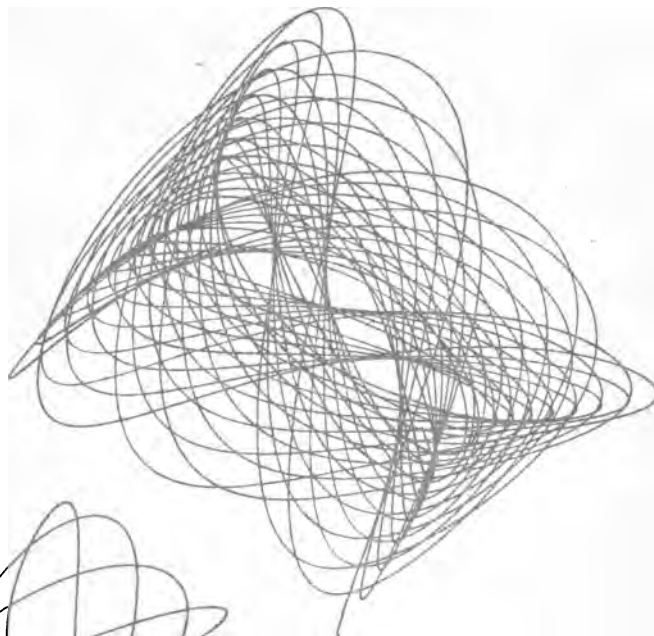


THE GREAT THIRD. Ratio 4:5.

*Number of Commas 8,9.*

# XIV

## THE PENDULOGRAPH.

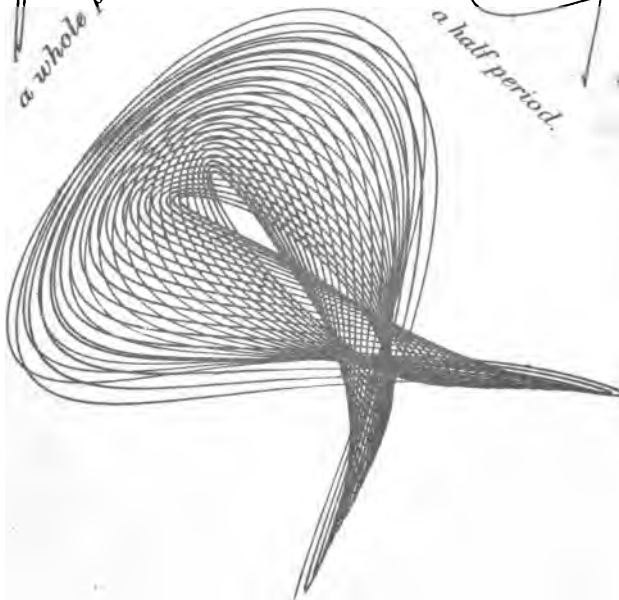
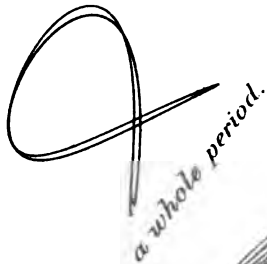


*a half period.*

THE SMALL SIXTH. Ratio 5:8.

*Number of Commas 5,5,8,9,9.*

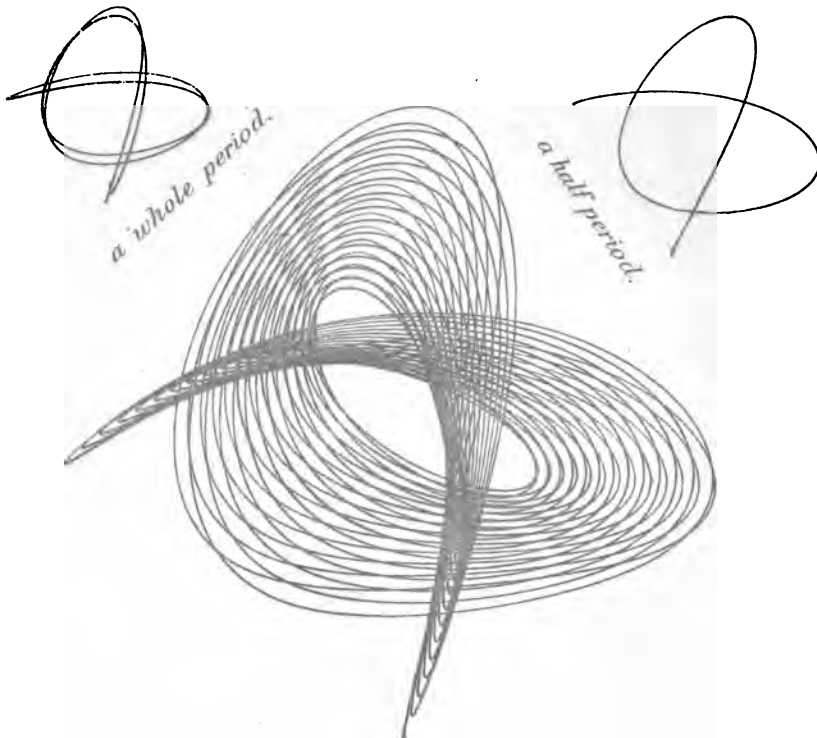
THE PENDULOGRAPH.



THE GREAT FIFTH. Ratio 2:3.

*Number of Commas 5,8,9,9.*

THE PENDULOGRAPH.

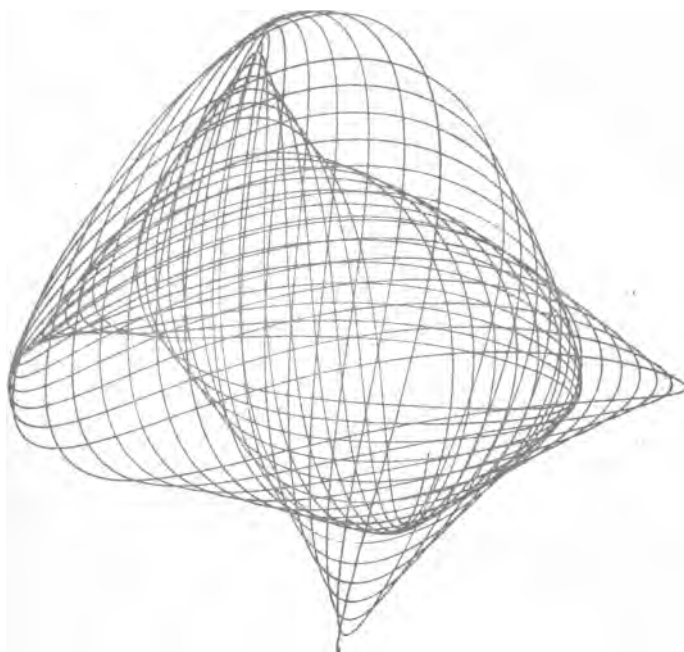


THE SMALL FOURTH. Ratio 3:4.

*Number of Commas 5, 8, 9.*

# XVII

## THE PENDULOGRAPH.

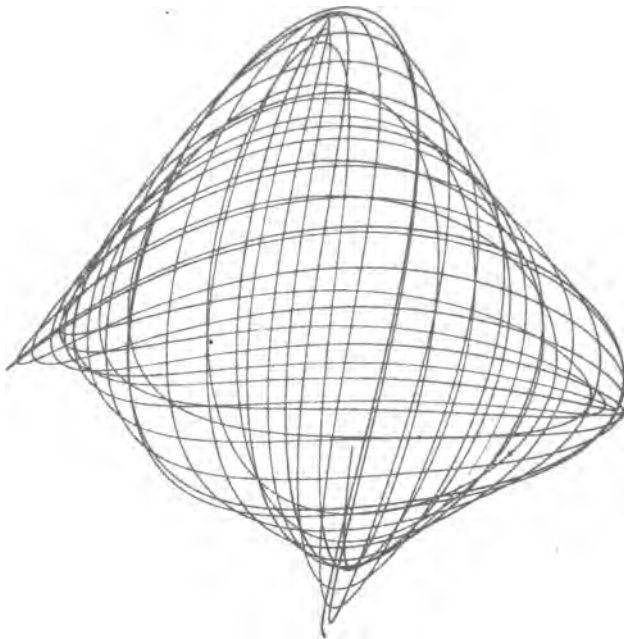


THE MEDIUM FIFTH. Ratio 27:40.

*Number of Commas 5, 8, 8, 9.*

# XVIII

## THE PENDULOGRAPH.

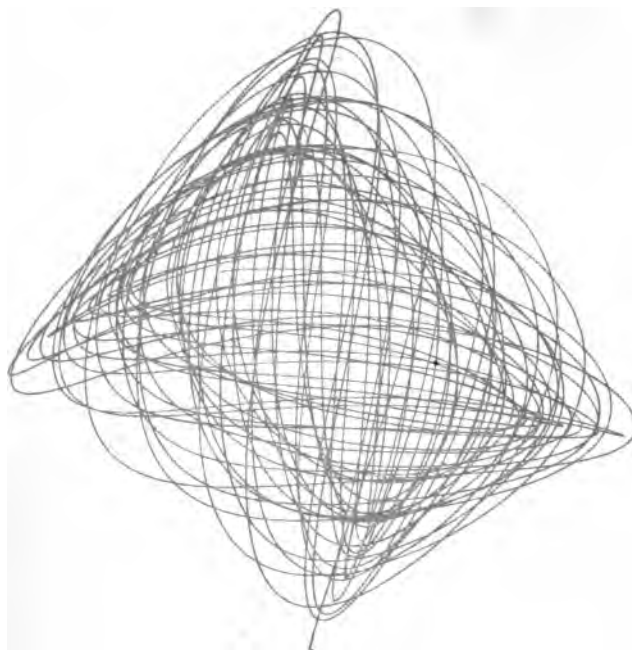


THE MEDIUM FOURTH. Ratio 20:27,

*Number of Commas 5,9,9.*

# XIX

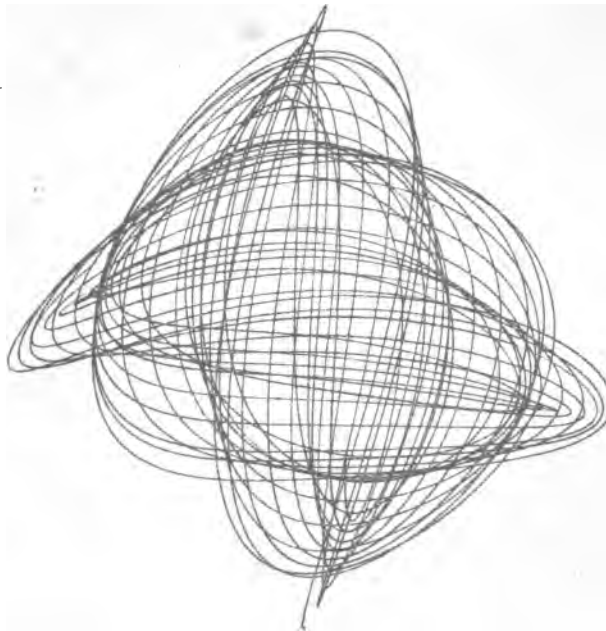
## THE PENDULOGRAPH.



THE SMALL FIFTH. Ratio 45 : 64.

*Number of Commas. 5,5,8,9.*

## THE PENDULOGRAPH.

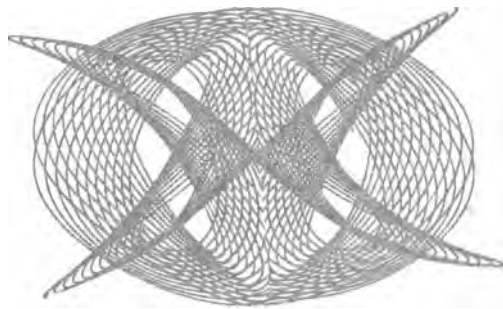


THE GREAT FOURTH Ratio 32:45.

*Number of Commas 8,9,9.*



THE



END.





